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THE VARIMAX CRITERION FOR ANALYTIC ROTATION IN FACTOR ANALYSIS*

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5237 citations as of 4/1/2016

- **Context (it was his PhD work in early 1955)**
- **Raw and Normal Varimax**
- **Follow-Ups**

Earlier Proposals for Rotation

"The first analytic criterion for determining psychologically interpretable factors was presented in 1953 by Carroll."

Comment: Kaiser hereby ignored earlier proposals to ***partially*** use analytic rotation criteria, along with graphical methods (e.g., Tucker, 1944). His view on these:

"In the light of later developments, Carroll´s criterion should probably be relegated to the limbo of 'near misses'; however, this does not detract from the fact that it was the first attempt to break away from an inflexible devotion to Thurstone's ambiguous, arbitrary, and mathematically unmanageable qualitative rules for his intuitively compelling notion of simple structure."(Kaiser, p. 188)

Existing Criteria are Shown to be Equivalent & Biased

Almost simultaneously, Saunders (1953), Neuhaus & Wrigley (1954), and Ferguson (1954) proposed differently formulated methods for orthogonal simple structure rotation. Kaiser elegantly showed them to be equivalent to Carroll's method. Following Neuhaus & Wrigley, he called it **quartimax**.

Then he pointed out that quartimax has a **bias**. He showed that quartimax was equivalent to maximizing the sum of **row-wise** variances of squared loadings. Hence:

"…its aim is to simplify the description of each row, or test, of the factor matrix. It is unconcerned with simplifying the columns, or factors, of the factor matrix (probably the most fundamental of all requirements for simple structure). The implication of this is that the quartimax criterion will often give a general factor." (o.c., p.190).

Varimax Looks at the Columns of the Factor Matrix

From this quote, almost automatically the suggestion emerged to **maximize the sum of *column-wise variances* of the squared loadings**; that is, the **Raw Varimax** criterion.

And for the criterion for all factors, define the maximum simplicity of a factor matrix as the maximization of

(8) $$v^* = \sum_s v_s^* = \sum_s \{[n \sum_i (a_{is}^2)^2 - (\sum_i a_{is}^2)^2]/n^2\},$$

the variance of squared loadings by columns rather than by rows.

He had presented the Raw Varimax criterion earlier at the 1955 APA conference. It also appeared in his unpublished PhD dissertation (1955).

Normalization on the Communalities

However, in an example Kaiser (1958) noted a **second bias**:

> "It seems reasonable to attribute the systematic bias […] to the divergent weights which implicitly are attached to the tests by their communalities. When one deals with fourth-power functions of factor loadings, a test with communality 0.6, for example, would tend to influence the rotations four times as much as a test whose communality was 0.3."

He then argued that the loadings should be normalized on their communalities. That leads to **Normal Varimax**:

for this suggestion.) The varimax criterion could then be rewritten as

$$v = \sum_s \{[n \sum_i (a_{is}^2/h_i^2)^2 - [\sum_i (a_{is}^2/h_i^2)]^2]/n^2\}, \quad (10)$$

where h_i^2 is the communality of the jth test. In contrast to (7) and (8), where

A More Fundamental Rationale Emerged

"**Saunders suggested** that the criterion should be applied to a factor matrix in which the rows had been normalized. This was crucial, for it gave better results subjectively, and, more importantly, **allowed me to prove a theorem** regarding the invariance of the varimax solution under the addition or subtraction of variables in a simple case.." (Kaiser, 1978)

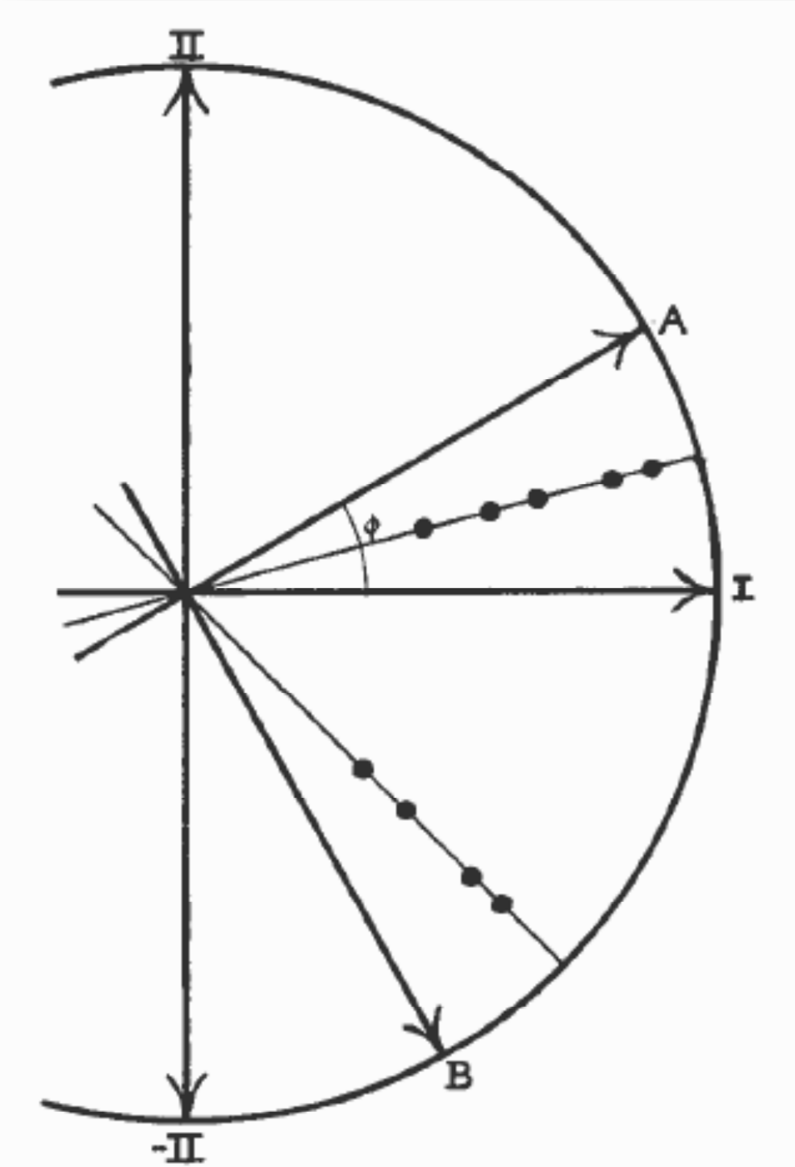


FIGURE 1

Invariance Property not as Strong as Suggested

For the case in which all tests load on only one factor, he showed mathematically that the **angle of rotation does not depend on the number of tests in each cluster**.

However, unfortunately

- It is not hard to show that this result does not generalize to more than two factors;
- It is simply a consequence of the fact that variances are considered of vectors with only two different values;
- It can be shown that the invariance property holds for optimizing any criterion using column-wise variances.

Comment Heiser: **Still I like this result!**

Oblique Rotation

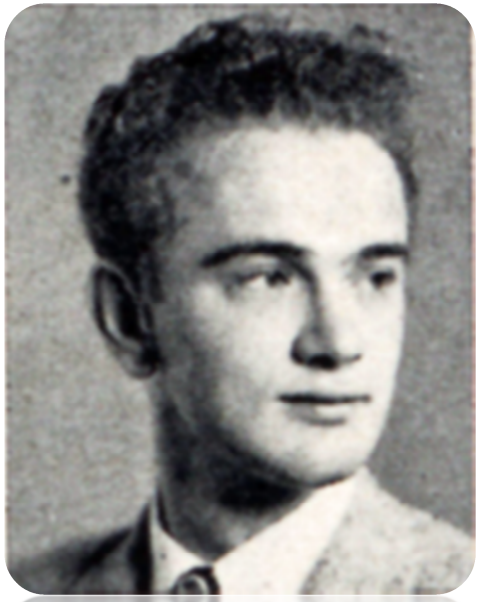
The paper essentially closes by a brief section on the oblique case.

In these 14 lines, Kaiser proposed an alternative criterion for oblique rotation (which was dubbed **covarimin** by Carroll (1957) even before Kaiser published his paper), and he suggested how to optimize it. It has by no means become as popular as **varimax**, but it is remarkable that in this paper on orthogonal simple structure rotation, almost in passing, he offered a new criterion for oblique rotation as well.

More Follow-Ups

The paper has a technical appendix that describes a planar rotation procedure for **iteratively maximizing** the Varimax criterion. Many future developments have taken Kaiser's Varimax as a starting point. We mention only two of them.

- One of the earliest such developments was Harris and Kaiser's (1964) successful "**orthoblique**" approach, where orthogonal varimax rotation was, in an ingenious way, used to find simple loadings for oblique factors.
- As another early follow up, the varimax solution was proposed for setting up a simple target for oblique target rotation: **Promax** method (Hendrickson & White, 1964).



On Kaiser's Personality

Henry Felix Kaiser (1927–1992) was born in Morristown, New Jersey, and educated in California, where he earned degrees at Berkeley in between periods of naval service during and after World War II. A specialist in psychological and educational statistics and measurement, Kaiser worked at the Universities of Illinois and Wisconsin before returning to Berkeley in 1968. He made several contributions to factor analysis, including varimax rotation (the subject of his PhD) and a measure for assessing sampling adequacy. Kaiser is remembered as an eccentric who spray-painted his shoes in unusual colors and listed ES (Eagle Scout) as his highest degree.

Quote from http://www.stata.com/manuals13/mvrotate.pdf